

**Class: XII Session: 2020-21**  
**Subject: Mathematics**  
**Sample Question Paper (Theory)**

**Time Allowed: 3 Hours**

**Maximum Marks: 80**

**General Instructions:**

1. This question paper contains two **parts A and B**. Each part is compulsory. Part A carries **24** marks and Part B carries **56** marks
2. **Part-A** has Objective Type Questions and **Part -B** has Descriptive Type Questions
3. Both Part A and Part B have choices.

**Part – A:**

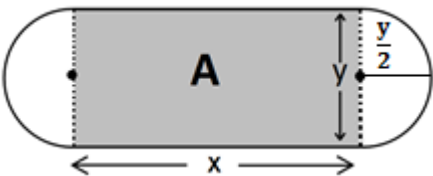
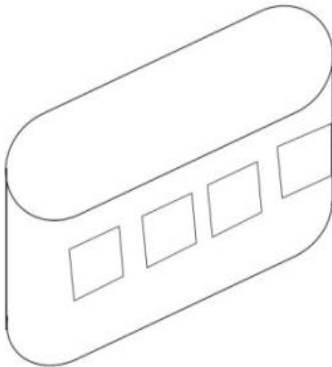
1. It consists of two sections- **I and II**.
2. Section **I** comprises of 16 very short answer type questions.
3. Section **II** contains **2** case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt **any 4 out of 5 MCQs**.

**Part – B:**


1. It consists of three sections- **III, IV and V**.
2. Section **III** comprises of 10 questions of **2 marks** each.
3. Section **IV** comprises of 7 questions of **3 marks** each.
4. Section **V** comprises of 3 questions of **5 marks** each.
5. Internal choice is provided in **3** questions of Section –III, **2** questions of Section-IV and **3** questions of Section-V. You have to attempt only one of the alternatives in all such questions.

<b>Sr. No.</b>	<b>Part – A</b>	<b>Marks</b>
	<b>Section I</b> <b>All questions are compulsory. In case of internal choices attempt any one.</b>	
1	Check whether the function $f: R \rightarrow R$ defined as $f(x) = x^3$ is one-one or not.  <b>OR</b>	1

	How many reflexive relations are possible in a set A whose $n(A) = 3$ .	1
2	A relation R in $S = \{1,2,3\}$ is defined as $R = \{(1, 1), (1, 2), (2, 2), (3, 3)\}$ . Which element(s) of relation R be removed to make R an equivalence relation?	1
3	A relation R in the set of real numbers $\mathbf{R}$ defined as $R = \{(a, b): \sqrt{a} = b\}$ is a function or not. Justify	1
	<b>OR</b>	
	An equivalence relation R in A divides it into equivalence classes $A_1, A_2, A_3$ . What is the value of $A_1 \cup A_2 \cup A_3$ and $A_1 \cap A_2 \cap A_3$	1
4	If A and B are matrices of order $3 \times n$ and $m \times 5$ respectively, then find the order of matrix $5A - 3B$ , given that it is defined.	1
5	Find the value of $A^2$ , where A is a $2 \times 2$ matrix whose elements are given by	1
	$a_{ij} = \begin{cases} 1 & \text{if } i \neq j \\ 0 & \text{if } i = j \end{cases}$	
	<b>OR</b>	
	Given that A is a square matrix of order $3 \times 3$ and $ A  = -4$ . Find $ \text{adj } A $	1
6	Let $A = [a_{ij}]$ be a square matrix of order $3 \times 3$ and $ A  = -7$ . Find the value of $a_{11} A_{21} + a_{12} A_{22} + a_{13} A_{23}$ where $A_{ij}$ is the cofactor of element $a_{ij}$	1
7	Find $\int e^x(1 - \cot x + \operatorname{cosec}^2 x) dx$	1
	<b>OR</b>	
	Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2 \sin x dx$	1
8	Find the area bounded by $y = x^2$ , the x-axis and the lines $x = -1$ and $x = 1$ .	1
9	How many arbitrary constants are there in the particular solution of the differential equation $\frac{dy}{dx} = -4xy^2$ ; $y(0) = 1$	1
	<b>OR</b>	
	For what value of n is the following a homogeneous differential equation:	1
	$\frac{dy}{dx} = \frac{x^3 - y^n}{x^2y + xy^2}$	
10	Find a unit vector in the direction opposite to $-\frac{3}{4} \hat{j}$	1
11	Find the area of the triangle whose two sides are represented by the vectors $2\hat{i}$ and $-3\hat{j}$ .	1

12	Find the angle between the unit vectors $\hat{a}$ and $\hat{b}$ , given that $ \hat{a} + \hat{b}  = 1$	1
13	Find the direction cosines of the normal to YZ plane?	1
14	Find the coordinates of the point where the line $\frac{x+3}{3} = \frac{y-1}{-1} = \frac{z-5}{-5}$ cuts the XY plane.	1
15	The probabilities of A and B solving a problem independently are $\frac{1}{3}$ and $\frac{1}{4}$ respectively. If both of them try to solve the problem independently, what is the probability that the problem is solved?	1
16	The probability that it will rain on any particular day is 50%. Find the probability that it rains only on first 4 days of the week.	1
	<b>Section II</b> <b>Both the Case study based questions are compulsory. Attempt any 4 sub parts from each question (17-21) and (22-26). Each question carries 1 mark</b>	
17	<p>An architect designs a building for a multi-national company. The floor consists of a rectangular region with semicircular ends having a perimeter of 200m as shown below:</p> <p style="text-align: center;"><b>Design of Floor</b></p>   <p style="text-align: center;"><b>Building</b></p> <p>Based on the above information answer the following:</p>	
	<p>(i) If x and y represents the length and breadth of the rectangular region, then the relation between the variables is</p> <p>a) <math>x + \pi y = 100</math>  b) <math>2x + \pi y = 200</math>  c) <math>\pi x + y = 50</math>  d) <math>x + y = 100</math></p>	

	<p>(ii) The area of the rectangular region A expressed as a function of x is</p> <p>a) <math>\frac{2}{\pi} (100x - x^2)</math></p> <p>b) <math>\frac{1}{\pi} (100x - x^2)</math></p> <p>c) <math>\frac{x}{\pi} (100 - x)</math></p> <p>d) <math>\pi y^2 + \frac{2}{\pi} (100x - x^2)</math></p>	1
	<p>(iii) The maximum value of area A is</p> <p>a) <math>\frac{\pi}{3200} m^2</math></p> <p>b) <math>\frac{3200}{\pi} m^2</math></p> <p>c) <math>\frac{5000}{\pi} m^2</math></p> <p>d) <math>\frac{1000}{\pi} m^2</math></p>	1
	<p>(iv) The CEO of the multi-national company is interested in maximizing the area of the whole floor including the semi-circular ends. For this to happen the value of x should be</p> <p>a) 0 m</p> <p>b) 30 m</p> <p>c) 50 m</p> <p>d) 80 m</p>	1
	<p>(v) The extra area generated if the area of the whole floor is maximized is :</p> <p>a) <math>\frac{3000}{\pi} m^2</math></p> <p>b) <math>\frac{5000}{\pi} m^2</math></p> <p>c) <math>\frac{7000}{\pi} m^2</math></p> <p>d) No change Both areas are equal</p>	1

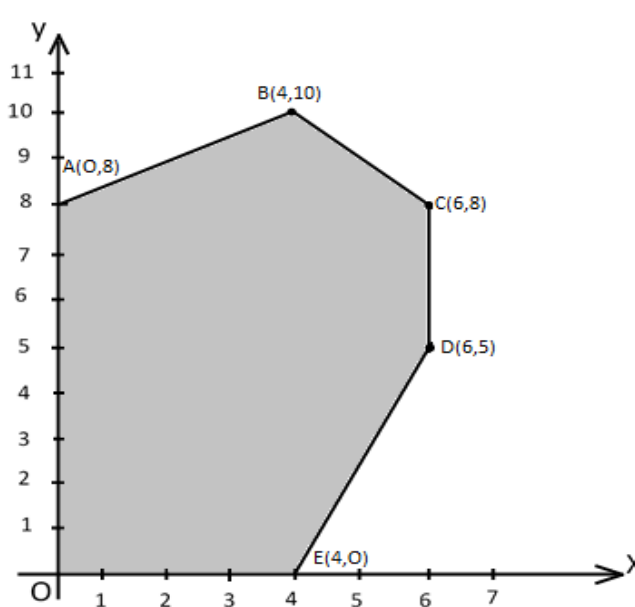
18	<p>In an office three employees Vinay, Sonia and Iqbal process incoming copies of a certain form. Vinay process 50% of the forms. Sonia processes 20% and Iqbal the remaining 30% of the forms. Vinay has an error rate of 0.06, Sonia has an error rate of 0.04 and Iqbal has an error rate of 0.03</p>  <p>Based on the above information answer the following:</p>	
	<p>(i) The conditional probability that an error is committed in processing given that Sonia processed the form is :</p> <p>a) 0.0210  b) 0.04  c) 0.47  d) 0.06</p>	1
	<p>(ii)The probability that Sonia processed the form and committed an error is :</p> <p>a) 0.005  b) 0.006  c) 0.008  d) 0.68</p>	1
	<p>(iii)The total probability of committing an error in processing the form is</p> <p>a) 0  b) 0.047  c) 0.234</p>	1

	d) 1	
	(iv)The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, the probability that the form is <b>NOT</b> processed by Vinay is :  a) 1 b) 30/47 c) 20/47 d) 17/47	1
	(v)Let A be the event of committing an error in processing the form and let E <sub>1</sub> , E <sub>2</sub> and E <sub>3</sub> be the events that Vinay, Sonia and Iqbal processed the form. The value of $\sum_{i=1}^3 P (E_i   A)$ is  a) 0 b) 0.03 c) 0.06 d) 1	1
	<b>Part – B</b>	
	<b>Section III</b>	
19	Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$ , $\frac{-3\pi}{2} < x < \frac{\pi}{2}$ in the simplest form.	2
20	If A is a square matrix of order 3 such that $A^2 = 2A$ , then find the value of  A .  <b>OR</b>  If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ , show that $A^2 - 5A + 7I = 0$ . Hence find $A^{-1}$ .	2  2
21	Find the value(s) of k so that the following function is continuous at $x = 0$	2

	$f(x) = \begin{cases} \frac{1 - \cos kx}{x \sin x} & \text{if } x \neq 0 \\ \frac{1}{2} & \text{if } x = 0 \end{cases}$	
22	Find the equation of the normal to the curve $y = x + \frac{1}{x}$ , $x > 0$ perpendicular to the line $3x - 4y = 7$ .	2
23	Find $\int \frac{1}{\cos^2 x (1 - \tan x)^2} dx$  <b>OR</b> Evaluate $\int_0^1 x(1 - x)^n dx$	2       2
24	Find the area of the region bounded by the parabola $y^2 = 8x$ and the line $x = 2$ .	2
25	Solve the following differential equation: $\frac{dy}{dx} = x^3 \operatorname{cosec} y$ , given that $y(0) = 0$ .	2
26	Find the area of the parallelogram whose one side and a diagonal are represented by coinitial vectors $\hat{i} - \hat{j} + \hat{k}$ and $4\hat{i} + 5\hat{k}$ respectively	2
27	Find the vector equation of the plane that passes through the point $(1,0,0)$ and contains the line $\vec{r} = \lambda \hat{j}$ .	2
28	A refrigerator box contains 2 milk chocolates and 4 dark chocolates. Two chocolates are drawn at random. Find the probability distribution of the number of milk chocolates. What is the most likely outcome?  <b>OR</b> Given that E and F are events such that $P(E) = 0.8$ , $P(F) = 0.7$ , $P(E \cap F) = 0.6$ . Find $P(\bar{E}   \bar{F})$	2       2
	<b>Section IV</b> <b>All questions are compulsory. In case of internal choices attempt any one.</b>	
29	Check whether the relation R in the set Z of integers defined as $R = \{(a, b) : a + b \text{ is "divisible by 2"}\}$ is reflexive, symmetric or transitive. Write the equivalence class containing 0 i.e. $[0]$ .	3
30	If $y = e^{x \sin^2 x} + (\sin x)^x$ , find $\frac{dy}{dx}$ .	3
31	Prove that the greatest integer function defined by $f(x) = [x]$ , $0 < x < 2$ is not differentiable at $x = 1$	3

	<b>OR</b>	
	If $x = a \sec \theta, y = b \tan \theta$ find $\frac{d^2y}{dx^2}$ at $x = \frac{\pi}{6}$	3
32	Find the intervals in which the function $f$ given by $f(x) = \tan x - 4x, x \in \left(0, \frac{\pi}{2}\right)$ is  a) strictly increasing <span style="margin-left: 200px;">b) strictly decreasing</span>	3
33	Find $\int \frac{x^2 + 1}{(x^2 + 2)(x^2 + 3)} dx$ .	3
34	Find the area of the region bounded by the curves $x^2 + y^2 = 4, y = \sqrt{3}x$ and $x$ - axis in the first quadrant	3
	<b>OR</b>	
	Find the area of the ellipse $x^2 + 9y^2 = 36$ using integration	3
35	Find the general solution of the following differential equation: $x dy - (y + 2x^2)dx = 0$	3
	<b>Section V</b>	
	<b>All questions are compulsory. In case of internal choices attempt any one.</b>	
36	If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$ , find $A^{-1}$ . Hence  Solve the system of equations;  $x - 2y = 10$ $2x - y - z = 8$ $-2y + z = 7$	5
	<b>OR</b>	
	Evaluate the product AB, where  $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$  Hence solve the system of linear equations  $x - y = 3$	5



	$2x + 3y + 4z = 17$ $y + 2z = 7$	
37	<p>Find the shortest distance between the lines</p> $\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$ <p>and <math>\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})</math></p> <p>If the lines intersect find their point of intersection</p> <p style="text-align: center;"><b>OR</b></p> <p>Find the foot of the perpendicular drawn from the point (-1, 3, -6) to the plane <math>2x + y - 2z + 5 = 0</math>. Also find the equation and length of the perpendicular.</p>	5
38	<p>Solve the following linear programming problem (L.P.P) graphically.</p> <p>Maximize <math>Z = x + 2y</math></p> <p>subject to constraints ;</p> $x + 2y \geq 100$ $2x - y \leq 0$ $2x + y \leq 200$ $x, y \geq 0$ <p style="text-align: center;"><b>OR</b></p> <p>The corner points of the feasible region determined by the system of linear constraints are as shown below:</p>  <p>Answer each of the following:</p> <p>(i) Let <math>Z = 3x - 4y</math> be the objective function. Find the maximum and minimum value of <math>Z</math> and also the corresponding points at which the maximum and minimum value occurs.</p>	5

	<p>(ii) Let <math>Z = px + qy</math>, where <math>p, q &gt; 0</math> be the objective function. Find the condition on <math>p</math> and <math>q</math> so that the maximum value of <math>Z</math> occurs at <math>B(4,10)</math> and <math>C(6,8)</math>. Also mention the number of optimal solutions in this case.</p>	
--	--	--

**Class: XII Session: 2020-21**

**Subject: Mathematics**

**Marking Scheme (Theory)**

Sr.No.	Objective type Question Section I	Marks
1	Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in R$ $\Rightarrow (x_1)^3 = (x_2)^3$ $\Rightarrow x_1 = x_2$ , Hence $f(x)$ is one – one  OR  $2^6$ reflexive relations	1          1
2	(1,2)	1
3	Since $\sqrt{a}$ is not defined for $a \in (-\infty, 0)$ $\therefore \sqrt{a} = b$ is not a function.  OR  $A_1 \cup A_2 \cup A_3 = A$ and $A_1 \cap A_2 \cap A_3 = \phi$	1       1
4	3x5	1
5	$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$  OR  $ \text{adj } A  = (-4)^{3-1} = 16$	1
6	0	1
7	$e^x(1 - \cot x) + C$  OR  $\because f(x)$ is an odd function  $\therefore \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2 \sin x \, dx = 0$	1          1
8	$A = 2 \int_0^1 x^2 \, dx = \frac{2}{3} [x^3]_0^1$ $= \frac{2}{3} \text{sq unit}$	1

9	0 <b>OR</b> 3	1  1
10	$\hat{j}$	1
11	$\frac{1}{2} 2\hat{i} \times (-3\hat{j})  = \frac{1}{2} -6\hat{k}  = 3 \text{ sq units}$	1
12	$ \hat{a} + \hat{b} ^2 = 1$ $\Rightarrow \hat{a}^2 + \hat{b}^2 + 2\hat{a} \cdot \hat{b} = 1$ $\Rightarrow 2\hat{a} \cdot \hat{b} = 1 - 1 - 1$ $\Rightarrow \hat{a} \cdot \hat{b} = \frac{-1}{2} \Rightarrow  \hat{a}  \hat{b}  \cos \theta = \frac{-1}{2} \Rightarrow \theta = \pi - \frac{\pi}{3}$ $\Rightarrow \theta = \frac{2\pi}{3}$	1
13	1,0,0	1
14	(0,0,0)	1
15	$1 - \frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$	1
16	$\left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^7$	1
<b>Section II</b>		
17(i)	(b)	1
17(ii)	(a)	1
17(iii)	(c)	1
17(iv)	(a)	1
17(v)	(d)	1
18(i)	(b)	1
18(ii)	(c)	1
18(iii)	(b)	1
18(iv)	(d)	1
18(v)	(d)	1
<b>Section III</b>		
19	$\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right) = \tan^{-1}\left[\frac{\sin\left(\frac{\pi}{2}-x\right)}{1-\cos\left(\frac{\pi}{2}-x\right)}\right]$ $\tan^{-1}\left[\frac{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right]$	$\frac{1}{2}$

	$\tan^{-1} \left[ \cot \left( \frac{\pi}{4} - \frac{x}{2} \right) \right] = \tan^{-1} \left[ \tan \frac{\pi}{2} - \left( \frac{\pi}{4} - \frac{x}{2} \right) \right]$ $\tan^{-1} \left[ \tan \left( \frac{\pi}{4} + \frac{x}{2} \right) \right] = \frac{\pi}{4} + \frac{x}{2}$	<p>1</p> <p><math>\frac{1}{2}</math></p>
20	$A^2 = 2A$ $\Rightarrow  AA  =  2A $ $\Rightarrow  A  A  = 8 A  \quad (\because  AB  =  A  B  \text{ and }  2A  = 2^3 A )$ $\Rightarrow  A ( A  - 8) = 0$ $\Rightarrow  A  = 0 \text{ or } 8$ <p style="text-align: center;"><b>OR</b></p> $A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$ $5A = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}, 7I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$ $\Rightarrow A^2 - 5A + 7I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$ $\Rightarrow A^{-1}(A^2 - 5A + 7I) = A^{-1}0$ $\Rightarrow A - 5I + 7A^{-1} = 0$ $\Rightarrow 7A^{-1} = 5I - A$ $\Rightarrow A^{-1} = \frac{1}{7} \left( \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \right)$ $\Rightarrow A^{-1} = \frac{1}{7} \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p>1</p> <p>1</p>
21	$\lim_{x \rightarrow 0} \frac{1 - \cos kx}{x \sin x} = \lim_{x \rightarrow 0} \frac{2 \sin^2 \left( \frac{kx}{2} \right)}{x \sin x}$ $= \lim_{x \rightarrow 0} \frac{2 \sin^2 \left( \frac{kx}{2} \right)}{\frac{x^2}{x \sin x}}$ $= \frac{\lim_{x \rightarrow 0} \frac{2 \sin^2 \left( \frac{kx}{2} \right)}{\left( \frac{kx}{2} \right)^2} \times \left( \frac{k}{2} \right)^2}{\lim_{x \rightarrow 0} \frac{\sin x}{x}} = \frac{2 \times 1 \times \frac{k^2}{4}}{1}$	<p><math>1 \frac{1}{2}</math></p>

	$\therefore f(x) \text{ is continuous at } x = 0$ $\therefore \lim_{x \rightarrow 0} f(x) = f(0)$ $\Rightarrow \frac{k^2}{2} = \frac{1}{2} \Rightarrow k^2 = 1 \Rightarrow k = \pm 1$	$\frac{1}{2}$
22	$y = x + \frac{1}{x} \Rightarrow \frac{dy}{dx} = 1 - \frac{1}{x^2}$ <p><math>\therefore</math> normal is perpendicular to <math>3x - 4y = 7</math>, <math>\therefore</math> tangent is parallel to it</p> $1 - \frac{1}{x^2} = \frac{3}{4} \Rightarrow x^2 = 4 \Rightarrow x = 2 \quad (\because x > 0)$ <p>when <math>x = 2</math>, <math>y = 2 + \frac{1}{2} = \frac{5}{2}</math></p> <p><math>\therefore</math> Equation of Normal : <math>y - \frac{5}{2} = -\frac{4}{3}(x - 2) \Rightarrow 8x + 6y = 31</math></p>	1  1
23	$I = \int \frac{1}{\cos^2 x (1 - \tan x)^2} dx$ <p>Put, <math>1 - \tan x = y</math></p> <p>So that, <math>-\sec^2 x dx = dy</math></p> $= \int \frac{-1 dy}{y^2} = - \int y^{-2} dy$ $= + \frac{1}{y} + c = \frac{1}{1 - \tan x} + c$ <p style="text-align: center;"><b>OR</b></p> $I = \int_0^1 x (1 - x)^n dx$ $I = \int_0^1 (1 - x)[1 - (1 - x)]^n dx$ $I = \int_0^1 (1 - x) x^n dx = \int_0^1 (x^n - x^{n+1}) dx$ $I = \left[ \frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2} \right]_0^1$ $I = \left[ \left( \frac{1}{n+1} - \frac{1}{n+2} \right) - 0 \right] = \frac{1}{(n+1)(n+2)}$	1  1  $\frac{1}{2}$  1  $\frac{1}{2}$
24	$\text{Area} = 2 \int_0^2 \sqrt{8x} dx$ $= 2 \times 2\sqrt{2} \int_0^2 x^{\frac{1}{2}} dx$	1

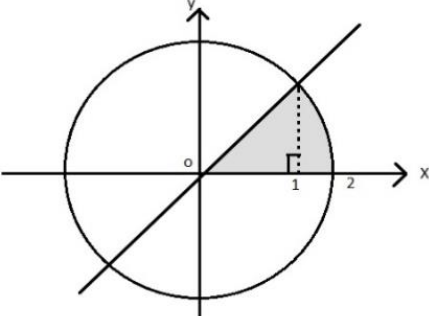
	$= 4\sqrt{2} \left[ \frac{2}{3} x^{\frac{3}{2}} \right]_0^2$ $= \frac{8}{3} \sqrt{2} \left[ 2^{\frac{3}{2}} - 0 \right] = \frac{8\sqrt{2}}{3} \times 2\sqrt{2}$ $= \frac{32}{3} \text{ sq units}$	$\frac{1}{2}$  $\frac{1}{2}$		
25	$\frac{dy}{dx} = x^3 \operatorname{cosec} y \quad ; \quad y(0) = 0$ $\int \frac{dy}{\operatorname{cosec} y} = \int x^3 dx$ $\int \sin y dy = \int x^3 dx$ $-\cos y = \frac{x^4}{4} + c$ $-1 = c \quad (\because y = 0, \text{ when } x = 0)$ $\cos y = 1 - \frac{x^4}{4}$	$\frac{1}{2}$  1  $\frac{1}{2}$		
26	<p>Let <math>\vec{a} = \hat{i} - \hat{j} + \hat{k}</math></p> <p><math>\vec{d} = 4\hat{i} + 5\hat{k}</math></p> <p><math>\therefore \vec{a} + \vec{b} = \vec{d} \therefore \vec{b} = \vec{d} - \vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}</math></p> $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 3 & 1 & 4 \end{vmatrix} = -5\hat{i} - 1\hat{j} + 4\hat{k}$ <p>Area of parallelogram = <math> \vec{a} \times \vec{b}  = \sqrt{25 + 1 + 16} = \sqrt{42} \text{ sq units}</math></p>	$\frac{1}{2}$  1  $\frac{1}{2}$		
27	<p>Let the normal vector to the plane be <math>\vec{n}</math></p> <p>Equation of the plane passing through (1,0,0), i.e., <math>\hat{i}</math> is  <math>(\vec{r} - \hat{i}) \cdot \vec{n} = 0 \dots\dots\dots(1)</math></p> <p><math>\therefore</math> plane (1) contains the line <math>\vec{r} = \vec{o} + \lambda \hat{j}</math></p> <p><math>\therefore \hat{i} \cdot \vec{n} = 0</math> and <math>\hat{j} \cdot \vec{n} = 0 \Rightarrow \vec{n} = \hat{k}</math></p> <p>Hence equation of the plane is <math>(\vec{r} - \hat{i}) \cdot \hat{k} = 0</math>  i.e., <math>\vec{r} \cdot \hat{k} = 0</math></p>	1         1		
28	<p>Let x denote the number of milk chocolates drawn</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding: 5px;">X</td> <td style="padding: 5px;">P(x)</td> </tr> </table>	X	P(x)	
X	P(x)			

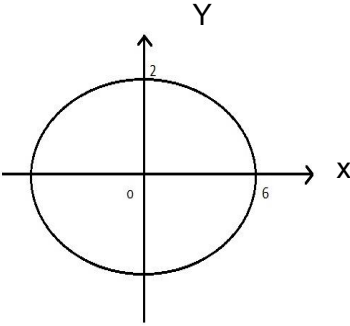
	<table border="1"> <tbody> <tr> <td>0</td> <td><math>\frac{4}{6} \times \frac{3}{5} = \frac{12}{30}</math></td> </tr> <tr> <td>1</td> <td><math>\left(\frac{2}{6} \times \frac{4}{5}\right) \times 2 = \frac{16}{30}</math></td> </tr> <tr> <td>2</td> <td><math>\frac{2}{6} \times \frac{1}{5} = \frac{2}{30}</math></td> </tr> </tbody> </table> <p>Most likely outcome is getting one chocolate of each type</p> <p style="text-align: center;"><b>OR</b></p> <p><math>P(\bar{E}   \bar{F}) = P\left(\frac{\bar{E} \cap \bar{F}}{P(\bar{F})} = \frac{P(\bar{E} \cup \bar{F})}{P(\bar{F})} = \frac{1 - P(E \cup F)}{1 - P(F)} \text{-----(1)}</math></p> <p>Now <math>P(E \cup F) = P(E) + P(F) - P(E \cap F)</math>  <math>= 0.8 + 0.7 - 0.6 = 0.9</math></p> <p>Substituting value of <math>P(E \cup F)</math> in (1)</p> <p><math>P(\bar{E}   \bar{F}) = \frac{1 - 0.9}{1 - 0.7} = \frac{0.1}{0.3} = \frac{1}{3}</math></p>	0	$\frac{4}{6} \times \frac{3}{5} = \frac{12}{30}$	1	$\left(\frac{2}{6} \times \frac{4}{5}\right) \times 2 = \frac{16}{30}$	2	$\frac{2}{6} \times \frac{1}{5} = \frac{2}{30}$	$1\frac{1}{2}$  $\frac{1}{2}$  1  $\frac{1}{2}$  $\frac{1}{2}$
0	$\frac{4}{6} \times \frac{3}{5} = \frac{12}{30}$							
1	$\left(\frac{2}{6} \times \frac{4}{5}\right) \times 2 = \frac{16}{30}$							
2	$\frac{2}{6} \times \frac{1}{5} = \frac{2}{30}$							
<b>Section IV</b>								
29	<p>(i) Reflexive : Since, <math>a+a=2a</math> which is even <math>\therefore (a,a) \in R \forall a \in Z</math> Hence R is reflexive</p> <p>(ii) Symmetric: If <math>(a,b) \in R</math>, then <math>a+b = 2\lambda \Rightarrow b+a = 2\lambda</math> <math>\Rightarrow (b,a) \in R</math>, Hence R is symmetric</p> <p>(iii) Transitive: If <math>(a,b) \in R</math> and <math>(b,c) \in R</math> then <math>a+b = 2\lambda</math>---(1) and <math>b+c = 2\mu</math> ---- (2) Adding (1) and (2) we get <math>a+2b+c=2(\lambda + \mu)</math> <math>\Rightarrow a+c=2(\lambda + \mu - b)</math> <math>\Rightarrow a+c=2k</math>, where <math>\lambda + \mu - b = k \Rightarrow (a,c) \in R</math> Hence R is transitive <math>[0] = \{\dots-4, -2, 0, 2, 4\dots\}</math></p>	$\frac{1}{2}$  1  $\frac{1}{2}$  $\frac{1}{2}$						
30	Let $u = e^{x \sin^2 x}$ and $v = (\sin x)^x$	$\frac{1}{2}$						



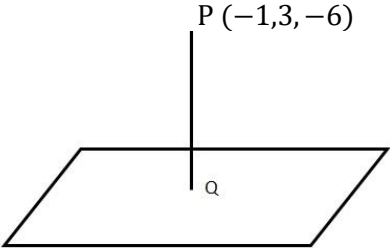
	<p>so that <math>y = u + v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}</math>-----(1)</p> <p>Now, <math>u = e^{x \sin^2 x}</math> , Differentiating both sides w.r.t. x, we get</p> $\Rightarrow \frac{du}{dx} = e^{x \sin^2 x} [x(\sin 2x) + \sin^2 x] \quad \text{----- (2)}$ <p>Also , <math>v = (\sin x)^x</math></p> $\Rightarrow \log v = x \log (\sin x)$ <p>Differentiating both sides w.r.t. x, we get</p> $\frac{1}{v} \frac{dv}{dx} = x \cot x + \log (\sin x)$ $\frac{dv}{dx} = (\sin x)^x [x \cot x + \log(\sin x)] \quad \text{----- (3)}$ <p>Substituting from – (2), – (3) in – (1) we get</p> $\frac{dy}{dx} = e^{x \sin^2 x} [x \sin 2x + \sin^2 x] + (\sin x)^x [x \cot x + \log(\sin x)]$	<p>1</p> <p>1</p> <p><math>\frac{1}{2}</math></p>
31	$\text{RHD} = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow 0} \frac{[1+h] - [1]}{h}$ $= \lim_{h \rightarrow 0} \frac{(1-1)}{h} = 0$ $\text{LHD} = \lim_{h \rightarrow 0} \frac{f(1-h) - f(1)}{-h} = \lim_{h \rightarrow 0} \frac{[1-h] - [1]}{-h} = \lim_{h \rightarrow 0} \frac{0-1}{-h}$ $= \lim_{h \rightarrow 0} \frac{1}{h} = \infty$ <p>Since, RHD <math>\neq</math> LHD Therefore <math>f(x)</math> is not differentiable at <math>x = 1</math></p> <p style="text-align: center;"><b>OR</b></p> $y = b \tan \theta \Rightarrow \frac{dy}{d\theta} = b \sec^2 \theta \dots (1)$ $x = a \sec \theta \Rightarrow \frac{dx}{d\theta} = a \sec \theta \tan \theta \dots (2)$	<p>1</p> <p>1</p> <p>1</p>

	$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{b \sec^2 \theta}{a \sec \theta \tan \theta} = \frac{b}{a} \operatorname{cosec} \theta$ <p><i>Differentiating both sides w.r.t. x, we get</i></p> $\frac{d^2y}{dx^2} = \frac{-b}{a} \operatorname{cosec} \theta \cot \theta \times \frac{d\theta}{dx}$ $= \frac{-b}{a} \operatorname{cosec} \theta \cot \theta \times \frac{1}{a \sec \theta \tan \theta} \quad [\text{using (2)}]$ $= \frac{-b}{a.a} \cot^3 \theta$ $\left. \frac{d^2y}{dx^2} \right _{\theta=\frac{\pi}{6}} = \frac{-b}{a} \left[ \cot \frac{\pi}{6} \right]^3 = \frac{-b}{a} (\sqrt{3})^3 = -\frac{3\sqrt{3}b}{a.a}$	$1\frac{1}{2}$          $\frac{1}{2}$
32	$f(x) = \tan x - 4x$ $f'(x) = \sec^2 x - 4$ <p>a) For <math>f(x)</math> to be strictly increasing</p> $f'(x) > 0$ $\Rightarrow \sec^2 x - 4 > 0$ $\Rightarrow \sec^2 x > 4$ $\Rightarrow \cos^2 x < \frac{1}{4} \Rightarrow \cos^2 x < \left(\frac{1}{2}\right)^2$ $\Rightarrow -\frac{1}{2} < \cos x < \frac{1}{2} \Rightarrow \frac{\pi}{3} < x < \frac{\pi}{2}$ <p>b) For <math>f(x)</math> to be strictly decreasing</p> $f'(x) < 0$ $\Rightarrow \sec^2 x - 4 < 0$ $\Rightarrow \sec^2 x < 4$ $\Rightarrow \cos^2 x > \frac{1}{4}$ $\Rightarrow \cos^2 x > \left(\frac{1}{2}\right)^2$ $\Rightarrow \cos x > \frac{1}{2} \left[ \because x \in \left(0, \frac{\pi}{2}\right) \right]$ $\Rightarrow 0 < x < \frac{\pi}{3}$	$\frac{1}{2}$          $1\frac{1}{2}$          $1$

<p>33</p>	<p>Put <math>x^2 = y</math> to make partial fractions</p> $\frac{x^2 + 1}{(x^2 + 2)(x^2 + 3)} = \frac{y + 1}{(y + 2)(y + 3)} = \frac{A}{y + 2} + \frac{B}{y + 3}$ $\Rightarrow y + 1 = A(y + 3) + B(y + 2) \dots \dots \dots (1)$ <p>Comparing coefficients of <math>y</math> and constant terms on both sides of (1) we get</p> $A + B = 1 \text{ and } 3A + 2B = 1$ <p>Solving, we get <math>A = -1, B = 2</math></p> $\int \frac{x^2 + 1}{(x^2 + 2)(x^2 + 3)} dx = \int \frac{-1}{x^2 + 2} dx + 2 \int \frac{1}{x^2 + 3} dx$ $= -\frac{1}{\sqrt{2}} \tan^{-1} \left( \frac{x}{\sqrt{2}} \right) + \frac{2}{\sqrt{3}} \tan^{-1} \left( \frac{x}{\sqrt{3}} \right) + C$	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p>1</p> <p>1</p>
<p>34</p>	<p>Solving <math>y = \sqrt{3}x</math> and <math>x^2 + y^2 = 4</math></p> <p>We get <math>x^2 + 3x^2 = 4</math></p> $\Rightarrow x^2 = 1 \Rightarrow x = 1$  <p>Required Area</p> $= \sqrt{3} \int_0^1 x dx + \int_1^2 \sqrt{2^2 - x^2} dx$ $= \frac{\sqrt{3}}{2} [x^2]_0^1 + \left[ \frac{x}{2} \sqrt{2^2 - x^2} + 2 \sin^{-1} \left( \frac{x}{2} \right) \right]_1^2$ $= \frac{\sqrt{3}}{2} + \left[ 2 \times \frac{\pi}{2} - \frac{\sqrt{3}}{2} - 2 \times \frac{\pi}{6} \right]$ $\frac{2\pi}{3} \text{ sq units}$ <p style="text-align: center;"><b>OR</b></p>	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p>1</p> <p><math>\frac{1}{2}</math></p>

	<p>Required Area = <math>\frac{4}{3} \int_0^6 \sqrt{6^2 - x^2} dx</math></p>  <p> <math display="block">= \frac{4}{3} \left[ \frac{x}{2} \sqrt{6^2 - x^2} + 18 \sin^{-1} \left( \frac{x}{6} \right) \right]_0^6</math> <math display="block">= \frac{4}{3} \left[ 18 \times \frac{\pi}{2} - 0 \right] = 12\pi \text{ sq units}</math> </p>	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p>1</p> <p>1</p>
35	<p>The given differential equation can be written as</p> $\frac{dy}{dx} = \frac{y + 2x^2}{x} \Rightarrow \frac{dy}{dx} - \frac{1}{x}y = 2x$ <p>Here <math>P = -\frac{1}{x}</math>, <math>Q = 2x</math></p> $\text{IF} = e^{\int P dx} = e^{-\int \frac{1}{x} dx} = e^{-\log x} = \frac{1}{x}$ <p>The solutions is :</p> $y \times \frac{1}{x} = \int \left( 2x \times \frac{1}{x} \right) dx$ $\Rightarrow \frac{y}{x} = 2x + c$ $\Rightarrow y = 2x^2 + cx$	<p><math>\frac{1}{2}</math></p> <p>1</p> <p>1</p> <p><math>\frac{1}{2}</math></p>
36	<p><math> A  = 1(-1 - 2) - 2(-2 - 0) = -3 + 4 = 1</math></p> <p>A is nonsingular, therefore <math>A^{-1}</math> exists</p> $\text{Adj } A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ $\Rightarrow A^{-1} = \frac{1}{ A } (\text{Adj } A) = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$	<p><math>\frac{1}{2}</math></p> <p><math>1\frac{1}{2}</math></p>

	<p>The given equations can be written as:</p> $\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$ <p>Which is of the form <math>A'X = B</math></p> $\Rightarrow X = (A')^{-1}B = (A^{-1})'B$ $\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$ $\Rightarrow x = 0, \quad y = -5, \quad z = -3$ <p style="text-align: center;"><b>OR</b></p> $AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ $= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$ $\Rightarrow AB = 6I$ $\Rightarrow A\left(\frac{1}{6}B\right) = I \Rightarrow A^{-1} = \frac{1}{6}(B)$ <p>The given equations can be written as</p> $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$ $AX = D, \text{ where } D = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$ $\Rightarrow X = A^{-1}D$ $\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix}$ $\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$ $x = 2, \quad y = -1, \quad z = 4$	<p style="text-align: center;"><math>\frac{1}{2}</math></p> <p style="text-align: center;">1</p> <p style="text-align: center;"><math>1\frac{1}{2}</math></p> <p style="text-align: center;"><math>1\frac{1}{2}</math></p> <p style="text-align: center;">1</p> <p style="text-align: center;">1</p> <p style="text-align: center;"><math>1\frac{1}{2}</math></p>
37	We have $a_1 = 3\hat{i} + 2\hat{j} - 4\hat{k}$ $b_1 = \hat{i} + 2\hat{j} + 2\hat{k}$	

	$a_2 = 5i - 2j$ $b_2 = 3i + 2j + 6k$ $\vec{a}_2 - \vec{a}_1 = 2i - 4j + 4k$ $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} i & j & k \\ 1 & 2 & 2 \\ 3 & 2 & 6 \end{vmatrix} = i(12 - 4) - j(6 - 6) + k(2 - 6)$ $\vec{b}_1 \times \vec{b}_2 = 8i + 0j - 4k = 8i - 4k$ $\therefore (\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1) = 16 - 16 = 0$ <p><math>\therefore</math> The lines are intersecting and the shortest distance between the lines is 0.</p> <p>Now for point of intersection</p> $3i + 2j - 4k + \lambda(i + 2j + 2k) = 5i - 2j + \mu(3i + 2j + 6k)$ $\Rightarrow 3 + \lambda = 5 + 3\mu \quad \text{--- -- (1)}$ $2 + 2\lambda = -2 + 2\mu \quad \text{--- -- (2)}$ $-4 + 2\lambda = 6\mu \quad \text{--- -- (3)}$ <p>Solving (1) and (2) we get, <math>\mu = -2</math> and <math>\lambda = -4</math></p> <p>Substituting in equation of line we get</p> $\vec{r} = 5i - 2j + (-2)(3i + 2j - 6k) = -i - 6j - 12k$ <p>Point of intersection is <math>(-1, -6, -12)</math></p> <p style="text-align: center;"><b>OR</b></p> <p>Let P be the given point and Q be the foot of the perpendicular.</p> <p>Equation of PQ <math>\frac{x+1}{2} = \frac{y-3}{1} = \frac{z+6}{-2} = \lambda</math></p> <div style="text-align: center;">  </div> <p>Let coordinates of Q be <math>(2\lambda - 1, \lambda + 3, -2\lambda - 6)</math></p> <p>Since Q lies in the plane <math>2x + y - 2z + 5 = 0</math></p> $\therefore 2(2\lambda - 1) + (\lambda + 3) - 2(-2\lambda - 6) + 5 = 0$ $\Rightarrow 4\lambda - 2 + \lambda + 3 + 4\lambda + 12 + 5 = 0$	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p><math>1\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p>
--	---	---

$$\Rightarrow 9\lambda + 18 = 0 \quad \Rightarrow \lambda = -2$$

$\therefore$  coordinates of Q are (-5, 1, -2)

$$\begin{aligned} \text{Length of the perpendicular} &= \sqrt{(-5 + 1)^2 + (1 - 3)^2 + (-2 + 6)^2} \\ &= 6 \text{ units} \end{aligned}$$

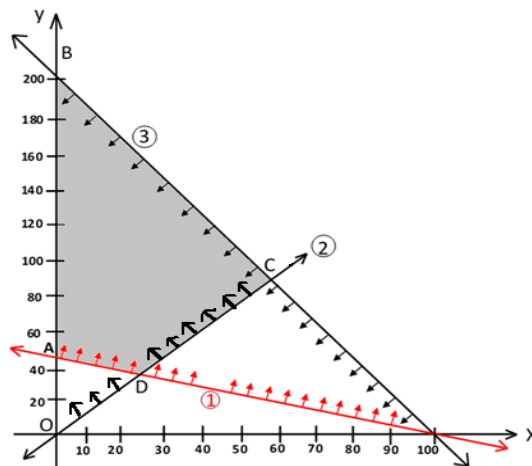
1  
1  
1

38

$$\text{Max } Z = 3x + y$$

Subject to

$$\begin{aligned} x + 2y &\geq 100 && \text{-----} && (1) \\ 2x - y &\leq 0 && \text{-----} && (2) \\ 2x + y &\leq 200 && \text{-----} && (3) \\ x &\geq 0, && y &\geq 0 \end{aligned}$$



3

Corner Points	$Z = 3x + y$
A (0, 50)	50
B (0, 200)	200
C (50, 100)	250
D (20, 40)	100

1

$$\text{Max } z = 250 \text{ at } x = 50, \quad y = 100$$

1

OR

(i)

Corner points	$Z = 3x - 4y$
O(0,0)	0
A(0,8)	-32
B(4,10)	-28
C(6,8)	-14
D(6,5)	-2
E(4,0)	12

$Max\ Z = 12\ at\ E(4,0)$

$Min\ Z = -32\ at\ A(0,8)$

(ii) Since maximum value of Z occurs at B(4,10) and C(6, 8)

$$\therefore 4p + 10q = 6p + 8q$$

$$\Rightarrow 2q = 2p$$

$$\Rightarrow p = q$$

Number of optimal solution are infinite

$1\frac{1}{2}$

1

2

$\frac{1}{2}$